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TITLE- Constant Look Angle Trajectories for the Visibility Phase of LM Descent

TM-70-2014-2

DATE- February 4, 1970

FILING CASE NO(S)- 310

AUTHOR(S)- W. B. Gevarter
S. B. Watson

FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- LM Descent, Trajectories, Quadratic Guidance

ABSTRACT

This memorandum presents a straightforward approach for generating constant look angle LM descent trajectories that satisfy the desired look angle, glide slope angle and certain boundary conditions. The resultant trajectories have constant thrust accelerations and vehicle attitudes. These simple trajectories provide good fuel usage and are readily mechanized by Quadratic Guidance. Higher order terms in the Quadratic Guidance expansion are available for terminal trajectory shaping.

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SUBJECT: Constant Look Angle Trajectories for the Visibility Phase of LM Descent - Case 310

DATE: February 4, 1970

FROM: W. B. Gevarter
S. B. Watson

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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

Studies of minimum fuel LM descent trajectories from high gate to low gate indicate that constant look angle trajectories may be a good approximation to these trajectories as well as having the obvious advantages for viewing. Figure 1 shows the associated vehicle-landing point geometry.

Reference 1 presents a method for solving for the Quadratic Guidance that would produce such trajectories. Examination of the resulting 13 simultaneous nonlinear equations indicates that the solution may be obtained directly. Thus, this memorandum reviews the basic equations and presents their solution. The solution is illustrated by the inclusion of several of the resulting trajectories.

2.0 BASIC EQUATIONS

From Figure 1, the equations of motion and the look angle of the LM in the vicinity of the landing site can be written directly as:

$$\ddot{X}_3 = -g + \frac{F}{m}\cos\alpha \qquad (2.1)$$

$$\ddot{X}_1 = -\frac{F}{m}\sin\alpha \qquad (2.2)$$

$$\gamma = \text{look angle in radians} = \frac{\pi}{2} - \alpha - \beta \qquad (2.3)$$

where

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TECHNICAL MEMORANDUM

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where

$$\frac{F}{m} = \frac{\text{Thrust}}{\text{mass}} = \text{thrust acceleration} \triangleq a(t) \tag{2.4}$$

$$\beta = \text{glideslope angle} = \tan^{-1}\left(\frac{X_3}{-X_1}\right) \tag{2.5}$$

<u>For Quadratic Guidance</u>, the X's have the form:

$$X_1(t) = \sum_{i=0}^{4} X_{1i} t^i \tag{2.6}$$

$$X_3(t) = \sum_{i=0}^{4} X_{3i} t^i \tag{2.7}$$

When Equations (2.6) and (2.7) are substituted in Equations (2.1) and (2.2), it becomes evident that for a constant look angle trajectory, an appropriate expansion for the thrust acceleration is:

$$a(t) = a_o + a_1 t + a_2 t^2 \tag{2.8}$$

Reference 1 solves the resulting differential equations, yielding 13 simultaneous nonlinear algebraic equation for the X and a coefficients. From examination of these equations* we find that a_1, a_2, X_{13}, X_{14}, X_{33}, X_{34} must all be zero. Thus, the problem reduces to:

---

*Equations (4.7) and (4.2) of Reference 1.

$$a = a_o \tag{2.9}$$

$$X_1(t) = X_{10} + X_{11}t + X_{12}t^2 \tag{2.10}$$

$$X_3(t) = X_{30} + X_{31}t + X_{32}t^2 \tag{2.11}$$

$$\ddot{X}_3 = 2X_{32} = -g + a_o \cos\alpha \tag{2.12}$$

$$\ddot{X}_1 = 2X_{12} = -a_o \sin\alpha \tag{2.13}$$

3.0 SOLUTION

From Equation (2.12), α is a constant. Thus, as α and γ are constants, Equation (2.3) indicates that β is a constant. Using Equations (2.10) and (2.11) for the X's in Equation (2.5) we have

$$X_{30}+X_{31}t+X_{32}t^2 = -\tan\beta \; [X_{10}+X_{11}t+X_{12}t^2] \tag{3.1}$$

As this is true for all t, the coefficients of t must be equal, yielding:

$$X_{30} = -\tan\beta \; X_{10} \tag{3.2}$$

$$X_{31} = -\tan\beta \; X_{11} \tag{3.3}$$

$$X_{32} = -\tan\beta \; X_{12} \tag{3.4}$$

From Equation (3.4) and Equations (2.12) and (2.13), we have

$$\tan\beta = \frac{X_{32}}{-X_{12}} = \frac{-1 + A\cos\alpha}{A\sin\alpha} \tag{3.5}$$

where we have defined A as:

$$A \triangleq \frac{a_o}{g} = \text{thrust acceleration in (moon) g's} \tag{3.6}$$

From Equation (3.5)

$$A = \frac{1}{\cos\alpha - \tan\beta\sin\alpha} \tag{3.7}$$

and from Equation (2.3)

$$\alpha° = 90° - (\beta° + \gamma°) \tag{3.8}$$

Thus

$$A = \frac{1}{\sin(\beta+\gamma) - \tan\beta\cos(\beta+\gamma)} \tag{3.9}$$

This is plotted in Figure 2.

4.0 FINDING THE COEFFICIENTS FOR A SPECIFIC TRAJECTORY

In this section we present a step-by-step method for find the coefficients of the X's to obtain a specific trajectory.

1. Choose a desired look angle, γ, and glide slope angle, β, and find A, the thrust acceleration in g's, from:

$$A = \frac{1}{\sin(\beta+\gamma) - \tan\beta \cos(\beta+\gamma)} \tag{4.1}$$

Figure 2 may be used as a check.

2. Calculate α, the vehicle angle from the vertical, from:

$$\alpha° = 90° - (\beta°+\gamma°) \tag{4.2}$$

3. Find X_{12} from:

$$X_{12} = \frac{-Ag \sin\alpha}{2} \tag{4.3}$$

4. Find X_{32} from:

$$X_{32} = -\tan\beta \; X_{12} \tag{4.4}$$

5. Choose a desired terminal position and velocity, e.g., $X_3(T)$ and $\dot{X}_3(T)$. Thus, (4.5)

$$X_3(T) = X_{30} + X_{31}T + X_{32}T^2 \tag{4.6}$$

$$\dot{X}_3(T) = X_{31} + 2X_{32}T \tag{4.7}$$

Using these equations, find the other parameters by choosing one of the following:

a. Select a time of flight, <u>T</u>, and find <u>X_{30}</u> and <u>X_{31}</u> from Equations (4.6) and (4.7) as:

$$X_{31} = \dot{X}_3(T) - 2X_{32}T \tag{4.8}$$

$$X_{30} = X_3(T) - X_{31}T - X_{32}T^2 \tag{4.9}$$

b. Select an initial condition such as $\dot{X}_1(0) \equiv$ <u>X_{11}</u> and find <u>X_{31}</u>, <u>T</u> and <u>X_{30}</u> from

$$X_{31} = -\tan\beta \ X_{11} \tag{4.10}$$

$$T = \frac{\dot{X}_3(T) - X_{31}}{2X_{32}} \tag{4.11}$$

$$X_{30} = X_3(T) - X_{31}T - X_{32}T^2 \tag{4.12}$$

6. Find <u>remaining X coefficients</u> from:

$$X_{3i} = -\tan\beta \ X_{1i} \tag{4.13}$$

7. Any <u>other quantities of interest</u> on the trajectory may now be obtained from:

$$X_1(t) = X_{10} + X_{11}t + X_{12}t^2 \tag{4.14}$$

$$X_3(t) = X_{30} + X_{31}t + X_{32}t^2 \tag{4.15}$$

5.0 QUADRATIC GUIDANCE

If we define a state vector:

$$\bar{x} = \begin{pmatrix} x_1 = X_1 \\ x_2 = \dot{X}_1 \\ x_3 = X_3 \\ x_4 = \dot{X}_3 \end{pmatrix} \tag{5.1}$$

then for Quadratic Guidance, the acceleration commands in the X_1 and X_3 directions are given by:

$$\dot{x}_{2c}(t) = \dot{x}_{2f} + \frac{6}{t-T}(x_{2f} + x_2) + \frac{12}{(t-T)^2}(x_{1f} - x_1) \tag{5.2}$$

$$\dot{x}_{4c}(t) = \dot{x}_{4f} + \frac{6}{t-T}(x_{4f} + x_4) + \frac{12}{(t-T)^2}(x_{3f} - x_3) \tag{5.3}$$

where the subscripts c denotes command, and f the value at the final time, T.

From Equations (4.14), (4.15) and (5.1) we can identify the quantities in the above equations as:

$$\dot{x}_{2f} = \ddot{X}_1(T) = 2X_{12} \tag{5.4}$$

$$x_{2f} = \dot{X}_1(T) = X_{11} + 2X_{12}T \tag{5.5}$$

$$x_{1f} = X_1(T) = X_{10} + X_{11}T + X_{12}T^2 \tag{5.6}$$

$$\dot{x}_{4f} = \ddot{X}_3(T) = 2X_{32} \tag{5.7}$$

$$x_{4f} = \dot{X}_3(T) = X_{31} + 2X_{32}T \tag{5.8}$$

$$x_{3f} = X_3(T) = X_{30} + X_{31}T + X_{32}T^2 \tag{5.9}$$

6.0 FUEL USED

The fuel used is given by

$$\text{Fuel} = M_o - M(T) \tag{6.1}$$

The mass is given by

$$M = M_o + \int_o^t \dot{M} dt \tag{6.2}$$

where

$$\dot{M} = -cF = -ca_o M \tag{6.3}$$

$$c = \frac{1}{32.2\ I_{sp}} \tag{6.4}$$

I_{sp} = specific fuel consumptions in lb thrust/lb fuel

Solving Equation (6.3) yields:

$$M(t) = M_o c^{-ca_o t} \tag{6.5}$$

Thus

$$\text{Fuel used in slugs} = M_o [1-e^{-ca_o t}] \tag{6.6}$$

$$\text{Thrust} = F(t) = a_o M_o [1-e^{-ca_o t}] \tag{6.7}$$

7.0 EXAMPLES: (For all examples, M_o = 588 slugs, and $c = 1.056 \times 10^{-4}$ slugs of fuel/lbs of thrust.) Using the method outlined in Section 4 and 6 we consider several examples. Examples 1 and 3 are chosen to have the same given conditions as those in Reference 2, except that the look angle is chosen to be the minimal acceptable value, and two values of glide slope, $\beta = 20°$ and $15°$, are considered to bracket the nominal trajectory of Reference 2. Examples 2 and 4 are designed to meet the vertical velocity restrictions at 500 ft.

1. Given:

$\gamma = 35°$

$\beta = 20°$

$X_3(T) = 149$ ft

$\dot{X}_3(T) = -3.4$ ft/sec

$\dot{X}_1(0) = X_{11} = 505$ ft/sec

Solution:

$\alpha = 35°$, $A = 1.638$g's, $T = 99.1$ secs,
Fuel used = 51.2 slugs

$X_{10} = -25{,}908$ ft	$X_{30} = 9{,}430$ ft
$X_{11} = 505.0$ ft/sec	$X_{31} = -183.8$ ft/sec
$X_{12} = -2.500$ ft/sec^2	$X_{32} = 0.9098$ ft/sec^2

2. Given:

$\gamma = 35°$

$\beta = 20°$

$X_3(T) = 500$ ft

$\dot{X}_3(T) = -20$ ft/sec

$\dot{X}_1(0) = X_{11} = 505$ ft/sec

Solution:

$\alpha = 35°$, $A = 1.638$g's, $T = 90.0$ secs,
Fuel used = 46.7 slugs

$X_{10} = -26{,}626$ ft $\quad X_{30} = 9{,}691$ ft

$X_{11} = 505.0$ ft/sec $\quad X_{31} = -183.8$ ft/sec

$X_{12} = -2.500$ ft/sec^2 $\quad X_{32} = 0.9098$ ft/sec^2

The results for examples 1 and 2 are plotted in Figures 3 and 4. It will be observed that for γ and β fixed, changing the boundary conditions simply shifted the hover (zero velocity) point on the trajectory, but did not change the trajectory itself.

3. Given:

$\gamma = 35°$

$\beta = 15°$

$X_3(T) = 149$ ft

$\dot{X}_3(T) = -3.4$ ft/sec

$\dot{X}_1(0) = X_{11} = 505$ ft/sec^2

Solution:

$\alpha = 40°$, $A = 1.684$g's, $T = 85.5$ secs,
Fuel used = 45.7 slugs

$X_{10} = -22{,}684$ ft $\quad X_{30} = 6078$ ft

$X_{11} = 505.0$ ft/sec $\quad X_{31} - 135.3$ ft/sec

$X_{12} = -2.879$ ft/sec^2 $\quad X_{32} = 0.7715$ ft/sec^2

4. Given:

$\gamma = 35°$

$\beta = 15°$

$X_3(T) = 500$ ft

$\dot{X}_3(T) = -20$ ft/sec

$\dot{X}_1(0) = X_{11} = 505$ ft/sec^2

Solution:

$\alpha = 40°$, $A = 1.684$g's, $T = 74.7$ sec,
Fuel used = 40.2 slugs

$X_{10} = -23{,}525$ ft $\qquad X_{30} = 6303$ ft

$X_{11} = 505.0$ ft/sec $\qquad X_{31} = -135.3$ ft/sec

$X_{12} = -2.879$ ft/sec^2 $\qquad X_{32} = 0.7715$ ft/sec^2

The results for Examples 3 and 4 are plotted in Figures 5 and 6. Again the difference between the two is simply a shift of the hover point. It should be noted that range at hover can be adjusted by simply offsetting the range, resulting in a slight departure from a constant look angle trajectory.

8.0 SUMMARY AND CONCLUSIONS

This memorandum presents a method for generating constant look angle trajectories that satisfy certain boundary conditions, the desired look angle and glide slope angle.

From consideration of the results of the examples and the analysis, the following conclusions can be made:

1. Though the boundary conditions are somewhat different, comparison with Reference 2 indicates that the fuel used for LM descent from high gate to low gate compares favorably with the constrained fuel optimum trajectories of Reference 2 for comparable times of flight. In fact, use of the Method of Steepest Ascent computer program described in References 3 and 4, indicates that for a given high gate, time of flight, and look angle constraints appropriate to a constant look angle trajectory, a constant look angle trajectory is fuel optimal.

2. Constant look angle trajectories are readily mechanized by Quadratic Guidance following the procedure given here.

3. Adjustment of range at hover can be made by offset aiming of range, resulting in a small variation from constant look angles.

4. For constant look angle trajectories, the vehicle pitch angle is constant, so that if no terminal guidance is employed, the vehicle will not be vertical at hover. However, if we consider the method given here as providing a basic constant look angle trajectory, the higher order coefficients (X_{13}, X_{14}, X_{33}, X_{34}) in the two expansions for the X's are available for terminal trajectory shaping.

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S. B. Watson

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Attachments
Figures 1-6

REFERENCES

1. W. B. Gevarter, "A Method for Determining Constant Look Angle Trajectories and Their Mechanization for the Visibility Phase of LM Descent - Case 310", Bellcomm Memorandum for File B69 11044, November 17, 1969.

2. W. B. Gevarter and G. M. Cauwels, "A Comparison of Some Constrained Fuel Optimum Trajectories with Those for Quadratic Guidance for LM Descent from High Gate to Low Gate", Bellcomm TM 70-2014-1, 1970.

3. Imanga, C. P., "Optimization of Nonlinear Systems, Method Of Steepest Ascent", Case 27540-3, BTL MM67-4264-8, August 10, 1967.

4. Gevarter, W. B., "Optimization of Terminal Descent", Case 310, Bellcomm TM 69-2014-6, September 25, 1969.

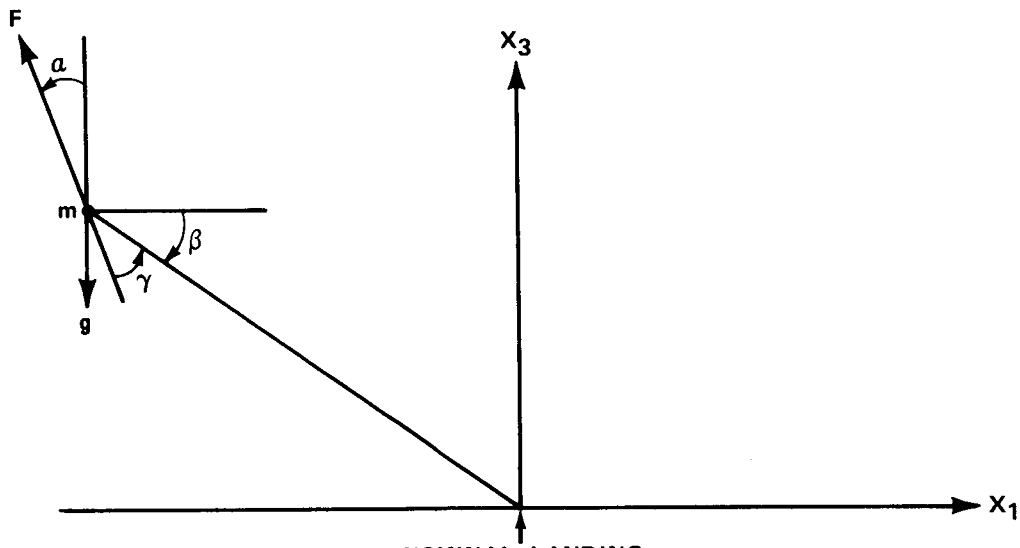

FIGURE 1 - VEHICLE AND LANDING POINT GEOMETRY

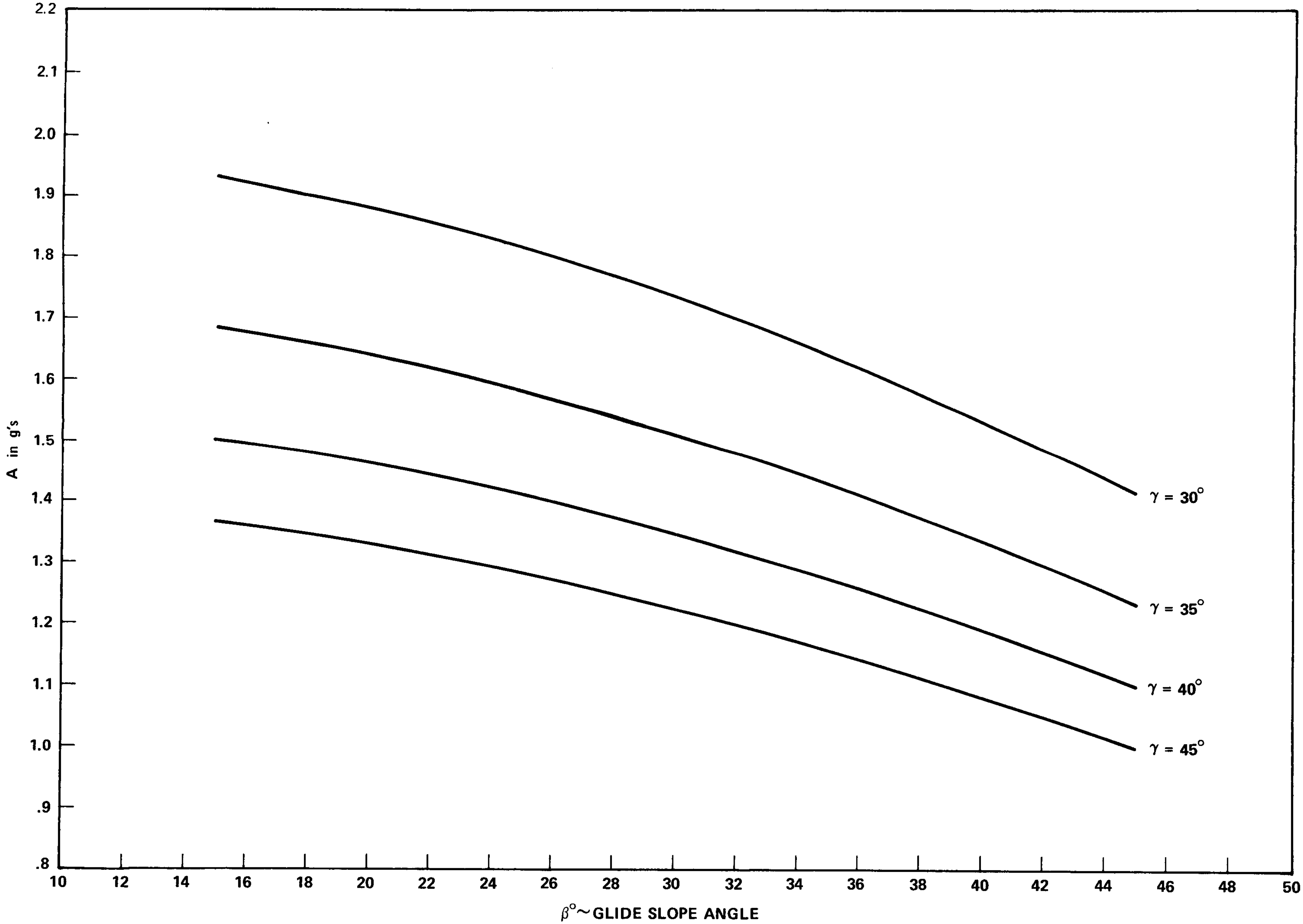

FIGURE 2 – THRUST ACCELERATION VS. GLIDE SLOPE AND LOOK ANGLE

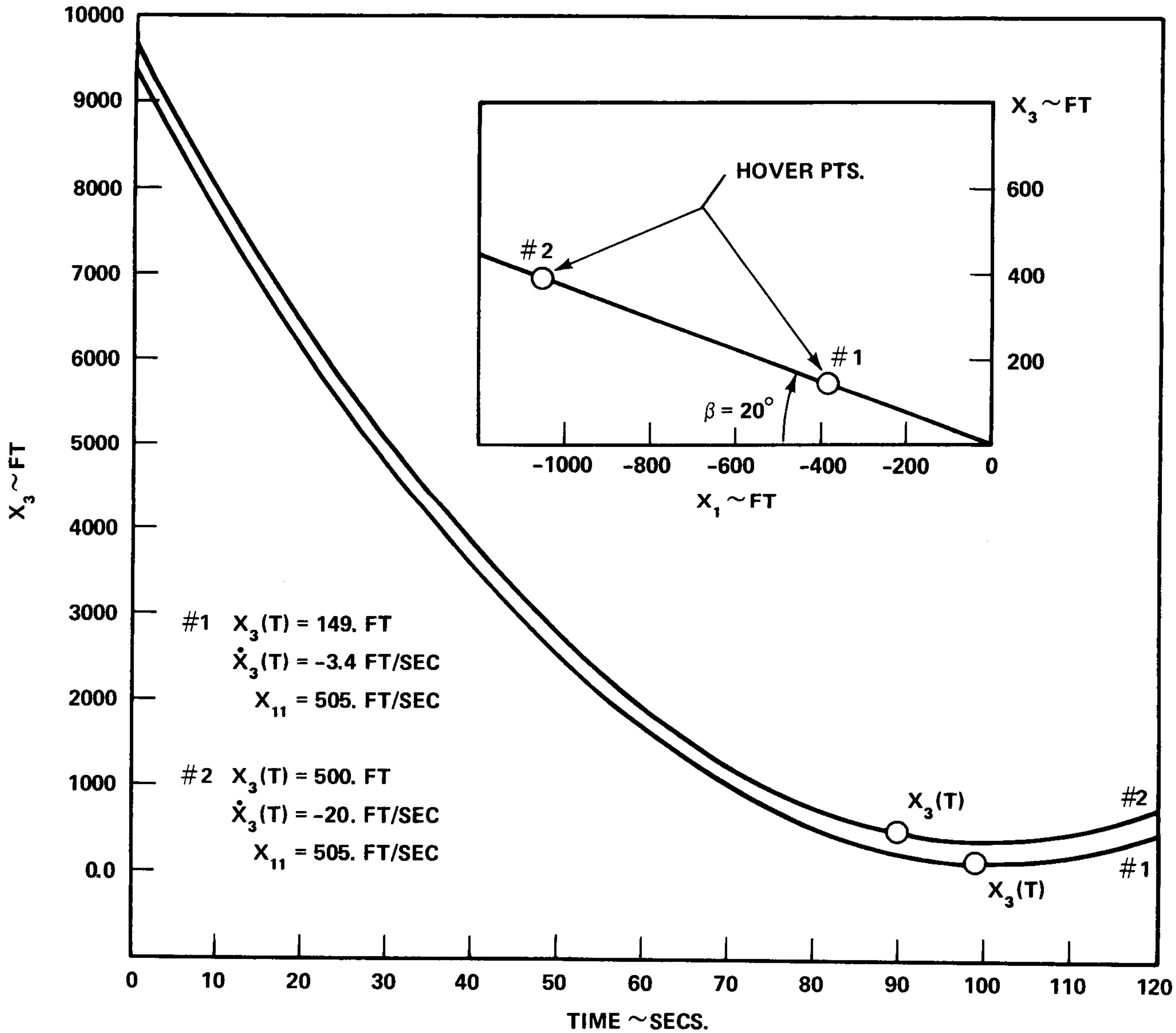

FIGURE 3 – ALTITUDE VS. TIME $\gamma = 35^\circ$, $\beta = 20^\circ$, ($\alpha = 35^\circ$)

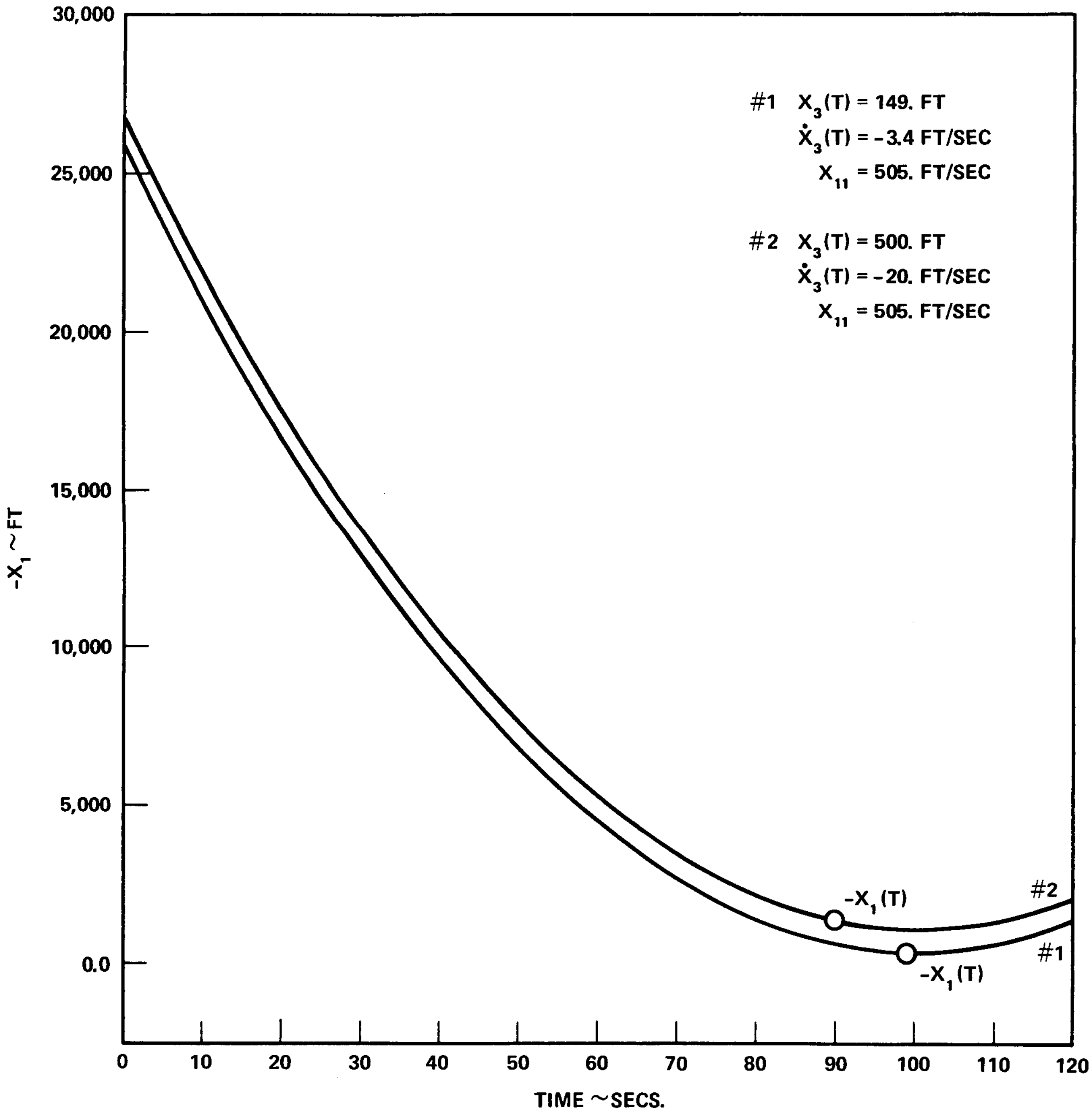

FIGURE 4 – RANGE VS. TIME $\gamma = 35^\circ$, $\beta = 20^\circ$, $(\alpha = 35^\circ)$

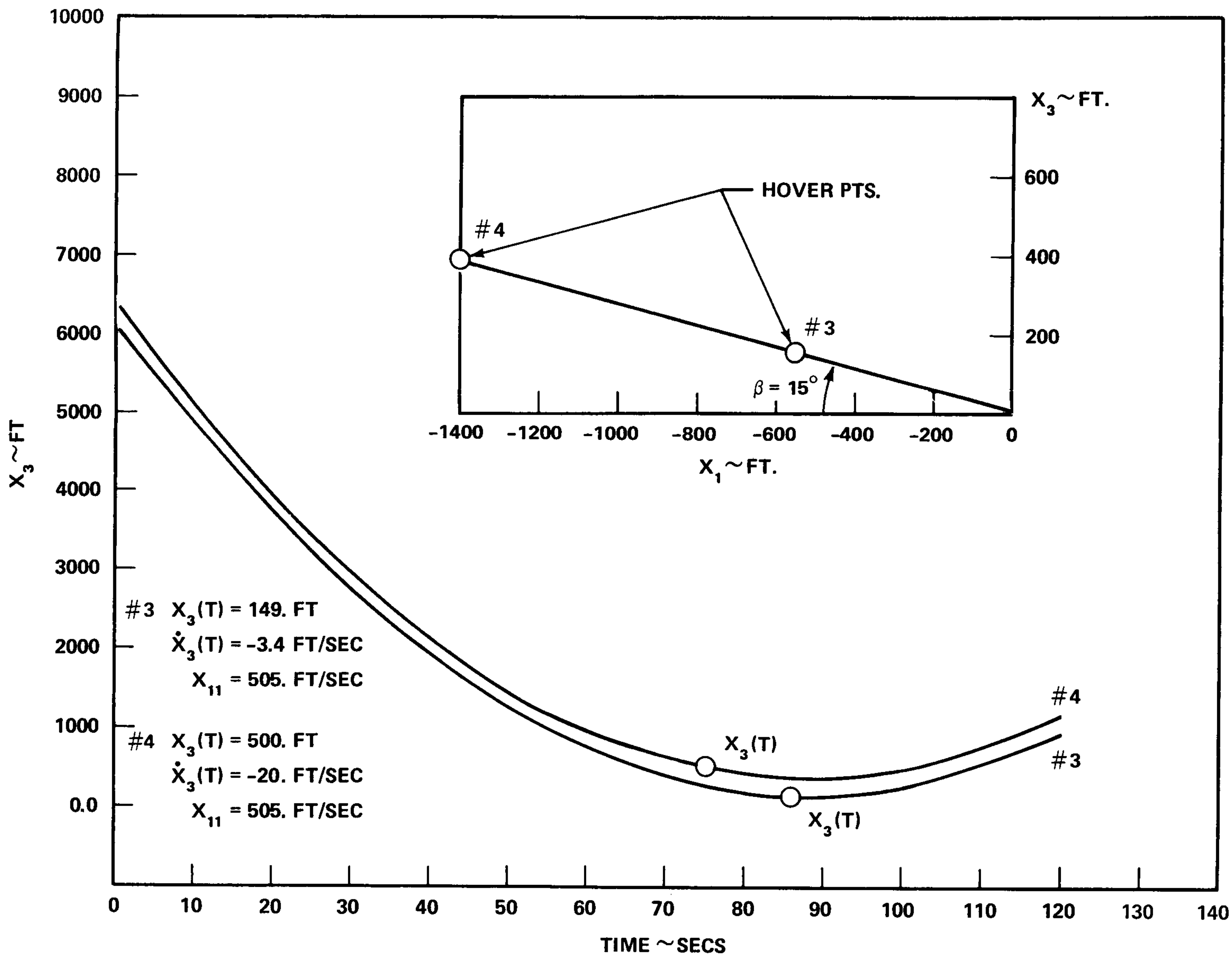

FIGURE 5 – ALTITUDE VS. TIME $\gamma = 35^\circ$, $\beta = 15^\circ$, $(\alpha = 40^\circ)$

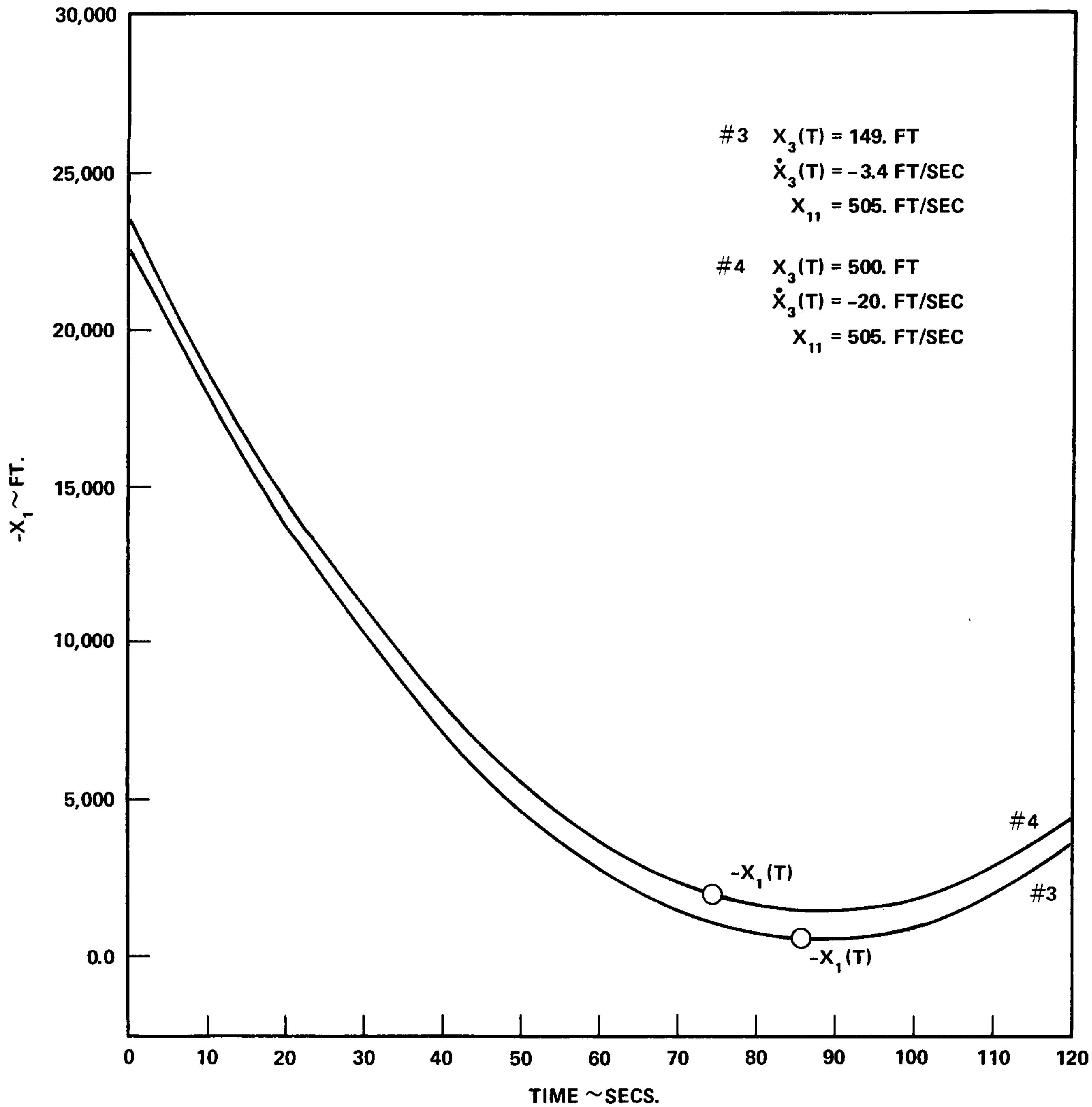

FIGURE 6 – RANGE VS. TIME $\gamma = 35^\circ$, $\beta = 15^\circ$, $(\alpha = 40^\circ)$